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COMMENT

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Abstract. It is pointed out, in connection with infinite conductivity in general relativistic magnetohydrodynamics, that in a region where the magnetic field lines are orthogonal to the vorticity vector there must be a nonzero conduction current. It is also shown that the relation $2\omega H = \epsilon$ derived by Yodzis assuming infinite conductivity, is also valid when the electric field vector satisfies Killing's equations.

In the standard book on general relativistic magnetohydrodynamics, Lichnerowicz (1967) shows that in the case of a perfect fluid with an infinite conductivity. $\sigma = \infty$, the electric current J^a, and thus the conduction current $\sigma E^a = J^a - \epsilon u^a$ being essentially finite, the electric field E^a must tend to zero. In this case J^a is not known, but it is only defined by the electromagnetic field equations.

A question arises: is it possible to indicate the cases when σE^a cannot be zero? It is the purpose of this communication to show that in a system composed of a plasma coupled to a frozen-in magnetic field there must be a nonzero conduction current when the magnetic field lines are orthogonal to the vorticity vector. We shall use the relation

$$D^b_{|b} - \alpha_a D^a + 2\omega H = \epsilon \tag{1}$$

obtained by Raychaudhuri and De (1970) (except for a change of signature and units), and Mason (1972). In the infinite-conductivity limit, $D^a = \lambda E^a = 0$ and hence (Yodzis (1971)

$$\epsilon = 2\omega H \tag{2}$$

where Yodzis's notation is used. It is known that the interstellar spaces in our Galaxy contain magnetic fields which, on the average, are oriented parallel to the Galactic disc (Parker 1970), and therefore, according to equation (2), in such a region the proper charge density ϵ is zero. It then follows from Ohm's law $J^a = \epsilon u^a + \sigma E^a$ (with $J^a \neq 0$ for a plasma), that the conduction current σE^a cannot be zero.

If the electrical conductivity σ is not infinite, and thus the electric field E^a is a non-null quantity, the proper charge density can still be zero when E^a is a Killing vector. Indeed, if E^a satisfies the equation

$$E_{a|b} + E_{b|a} = 0 \tag{3}$$

we have

$$D^a_{|a} = 0, \qquad \text{and} \qquad \alpha_a D^a = 0 \tag{4}$$

after having made use of obvious equation $u^a D_a = 0$. Substituting equations (4) into equation (1) we obtain (2) and its consequences as claimed.

References

Lichnerowicz A 1967 *Relativistic Hydrodynamics and Magnetohydrodynamics* (New York: Benjamin) p93
Mason D P 1972 *J. Phys.* A: *Gen. Phys.* **5** L91–2
Parker E N 1970 *Astrophys. J.* **160** 383–404
Raychaudhuri A K and De U K 1970 *J. Phys.* A: *Gen. Phys.* **3** 263–8
Yodzis P 1971 *Phys. Rev.* D **3** 2941–5